

Optimal Allocation of Physical and Skills Capital in Services Production

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Abstract

”Software/ Skills” capital differs from usual physical capital (or hardware) in the sense that it is non-rival and can be **replicated** at a cost (e.g. patent fee or training costs). A basic model of production is developed which involves production sector and training or replication sector (which produces skills).

Using a 2 period production model, the paper finds that in sectors where the objective is output maximization (e.g. government services or health care) -

There exists an optimal ratio of investment in physical capital and investment in skills-capital depending on the state of technology and already existing stocks.

*In a capital-rich economy, a higher proportion of skills is allocated to production sector and a higher proportion of investment is allocated to **training sector** compared to capital-scarce economy .*

During high-investment periods, a higher share of investment goes to physical capital while a lower share of skills goes into production sector (compared to low-investment period).

Initial stock of skills, does not have any affect on these allocation-ratios.

1 Introduction

The trend of scientific innovations in the production context is very much clear. Most of us will agree that production technologies have shown a systematic movement from Agricultural to Manufacturing to Services.

Is there some logic behind this movement in production process? And if so, can we explain this process is a result of rational consumer optimization?

The answer is YES and I try to explain this process a bit while discussing the production side of service sector.

The idea is that optimization requires minimum use of resources per unit produced. Since resources are limited the success of technology lies in finding ways to produce more using the same amount. But a parallel way will be to develop technologies in which resources (factors of production) can themselves be produced.¹

This will generate a movement from consuming very few simple goods towards consuming a lot of complex/ multi-stage goods (generated after lots of intermediate steps, which themselves are results of production processes).

I extend the consumer side of service sector model to introduce production of these services. Then I try to use this setup to get insights into optimal ratios of skills allocation and investment allocation in the two sectors (production of final services and production of more skills).

2 Evolution of Production Process

Economics is science of allocating scarce resources with the objective of generating maximum welfare. One can directly consume these resources OR can use these resources as factors of production. Put simply, if the production process *generates* more (in number or in value) resources than it *uses* then it will make consumer better off than consuming the resources directly. So the success of production technology lies in generating maximum output for given amount of input, since the input is constrained by the total resources available. This is the case with Agricultural production where land is the main factor of production.

If in place of consuming the output of production process directly we use it as input to a further production process (which, as above, should generate some extra output) then the constraint on the resource is no longer relevant. The decision on how much of the output to consume and how much to use at

¹This is another way of saying that "Use some of the output to produce more stuff rather than consuming it" (which is a crude definition of Capital).

input next-step-production becomes important. In this way economy keeps on expanding the resource constraint in each step. Hence production technology is augmented by this accumulation process. This kind of logic is valid for Manufacturing industry where capital plays very important role.

The next logical step would be for economy to use some of the output to produce the inputs itself (earlier this output was being used as input, now it is used for *producing the inputs*). In this way the resource constraint will no longer be restricted to expand linearly in each step (as a result of partial accumulation of output as input-resources). But instead the resource constraint expands to the order of this **Input-production function**. Notice that this input-production function is different from the concept of intermediate-goods (which is a part of accumulation process where *all* the output of one step is used as input to next step). This production/ replication of inputs greatly enhances the total production, since inputs produced can then be used to produce goods and services for consumption. This is more applicable to service industry which relies on *Skills* as input which are generated as a result of another production process namely education or training.

Based on above discussion, following can be thought of as being the logical explanation for the movement in production from Agricultural to Manufacturing to Services.

Sector	Agricultural	Manufacturing	Services
Main Factor of Production	Land	Capital	Skills
Scarcity	Fixed	Can be Accumulated.	Can be Replicated.
Constraint Functional Form	L=Constant	K=Linear fn.	S= Exponential fn.

By switching from using a "Fixed" factor to something that can be "Accumulated" the economy is reducing the scarcity. Similarly, economy is moving one step further when it shifts to a factor of production that can be produced or "Replicated".

2.1 A Simple Numerical Example:

Consider a simple economy with endowment of say 10 inputs each period. The production function is such that it doubles the input. Half of that production is used for accumulation or input-production and the other half is consumed.

FIXED	ACCUMULATION	REPLICATION/ INPUT- PRODUCTION
Period 1: $f(10) = 20$	Period 1: $f(10) = 20$; 10 units are accumulated and used as additional input next period	Period 1: $f(10) = 20$; 10 units are used to produce input generating 20 extra units of inputs for next period.
Period 2 : $f(10) = 20$	Period 2: $f(10+10) = 40$; 20 units are accumulated.	Period 2: $f(10+20) = 60$; 30 units used for input production.
Period 3: $f(10) = 20$	Period 3: $f(10+20) = 60$	Period 3: $f(10+60) = 140$
...
Consumption in Nth Period: $2 * 10$	Consumption in Nth Period: $N * 10$	Consumption in Nth Period: $(2^N - 1) * 10$

3 Production of Services

Here are few concepts required to setup a production model for service sector.

1. **Hardware (K)** : It is rival and can be *accumulated* over time.

$$K_{t+1} = K_t * (1 - \delta) + I_K$$

2. **Software (L)** : It is non-rival and can be *replicated* at a cost.

$$L_{t+1} = L_t(1 + \frac{I_L}{C_L}) \text{ where } C_L \text{ is the cost of replication.} \quad (1)$$

Service is defined as completion of a *TRANSACTION*. The output is measured as no. of transactions.

Production of services takes place using both Software and Hardware, but marginal production only requires the use of software. The hardware determines the maximum number of transactions that can be performed.

3.1 An Inter-temporal model of Production

Consider 2 period model of production in an economy with capital stock K_0 and skills stock L_0 . Suppose initial investment endowment I_0 can be used either to augment the capital stock or the skills stock in the next period.

The stock of skills L_0 can be used either to produce services or to further augment the stock of skills.²

Let L_s be the skills allotted to production and $(L_0 - L_s)$ allotted to skills-replication. Also, let I_K be the investment endowment allocated for capital and I_L allocated for skills sector. Then in second period -

$$K_1 = K_0 * (1 - \delta) + I_K \quad (2a)$$

$$L_1 = L_0 + (L_0 - L_s) * \left(\frac{I_L}{C_L}\right)^3 \quad (2b)$$

$$I_K + I_L = I_0 \quad (2c)$$

where C_L is the cost of replicating one unit of skills.

Now consider economy's problem as that of maximizing discounted sum of output given initial stocks and investment endowments. This type of objective is justified in Government services context where objective is output maximization and not the profit. In sectors like health care where it is really hard to measure the value of output (e.g. lives saved or patients cured), output maximization seems like a reasonable objective.

The problem can be written as ⁴ -

$$\max_{L_s, I_K} [f(K_0, L_0) + \beta f(K_1, L_1)] \quad (3)$$

²An example of setup like this will be *Call Center* industry in India. India has number of English-speaking people, they can either work in call-centers or work in training other peoples to speak English.

³It is assumed that economy has enough people to learn these skills i.e. $Pop > \frac{I_L}{C_L}$

⁴Here discussion does not consider what happens to the end-of-period capital stock. Because it is also not considering what happens to end-of-period skills stock.

subject to constraints 2a, 2b and 2c.

This gives following optimality conditions -

$$f'_K(K_1, L_1) = \frac{(L_0 - L_s)}{C_L} * f'_L(K_1, L_1) \quad (4a)$$

$$f'_L(K_0, L_0) = \beta * \frac{(I_0 - I_K)}{C_L} * f'_L(K_1, L_1) \quad (4b)$$

- Equation 4a is the optimal investment decision rule, equating the return on two types of investments; I_K and I_L (return on I_L is the product of amount of skills generated using the investments and the marginal product of skill)
- Equation 4b is the optimal skills allocation; which equates the return in production and skills-replication.

4 Optimal Ratios Result

The optimal allocation of skills and investment endowments give -

1. Skills Intensity of Investment:

$$R_I^* = (1 - \frac{I_K^*}{I_0})$$

This is the share of investment going in replication/ training of skills sector (the rest goes towards physical capital).

2. Replication/Training Intensity of Skills:

$$R_S^* = (1 - \frac{L_S^*}{L_0})$$

This is the share of skills that is allotted in training sector for skills production.

This production function $f(K, L)$ is assumed to follow usual properties (concavity in K and L etc.).

Using optimality conditions 4a and 4b, some interesting comparative statistics about optimal ratios can be derived.⁵

⁵In some derivations below, $f(K, L)$ is also assumed to be homogeneous of degree 1 in K and L.

4.1 Relationship with Capital Stock

If initial endowment of **capital stock goes up** ; i.e. $K_0 \uparrow$:

Primary effect -

$f'_L(K_0, L_0) \uparrow$ and $f'_K(K_0, L_0) \downarrow \Rightarrow I_K \downarrow$ (from 4b)

Secondary effect -

As $I_L \uparrow$, $f'_L(K_1, L_1)$ begins to \downarrow due to increase in L_1 . But to make 4a hold $L_s \uparrow$. This makes $f'_L(K_1, L_1)$ to \downarrow (or stop \uparrow)

In capital-abundant economies, the ratio of the skilled-force allocated in production sector to skill-training sector is higher (compared to capital-scarce economies).

Similarly, *In capital-scarce economies, higher proportion of skills are allocated to the training sector(compared to capital-rich economies).*

What happens to the optimal investment allocation ratio?

As capital stock grows, a higher ratio of total investment is allocated in the skills-training sector.

Economic Intuition:

Here is how the whole process works -

- Increase in capital stock, decreases the return on physical-capital investment and increases the return on skills-capital investment. This leads to *increase in I_L or decrease in I_K .*
- With higher investment in training sector, lower amount of skills is needed (to generate the same amount of total skills-stock in next period). Hence L_s *increases.*

4.2 Relationship with Total Investment

If $I_0 \uparrow$, then as a direct effect (from 4b) $f'_L(K_0, L_0) \uparrow$. This means that $L_s \downarrow$. Using 4a, it is clear that ratio of return on physical-capital to return on skills-capital \uparrow . Hence it will lead to $\frac{I_K}{I_0} \uparrow$.

Economic Intuition -

- Increase in total available investment makes the return on labor next period less attractive compared to current period production (since some of this increased investment will result in $I_L \uparrow \Rightarrow L_1 \uparrow$). Hence less labor is allocated for skills-replication/ training sector.
- This decrease makes return on physical-capital next period better compared to return on skills-capital. Hence a higher share of investment is invested as physical capital.

In high-investment times, more of the skills are allocated to production and higher share of investment is invested in physical capital (compared to low-investment times).

4.3 Relationship with Skills Stock

If $L_0 \uparrow$, then nothing changes on the RHS of 4b. This means $\frac{f'_L(K_0, L_0)}{f'_L(K_1, L_1)}$ remains same.

This would imply that since the wedge between returns on skills allocated to production and to training/ replication does not change, the optimal $\frac{L_s}{L_0}$ will remain the same ⁶ (the value of L_s will change).

From 4a, $\frac{f'_K(K_1, L_1)}{f'_L(K_1, L_1)} \uparrow$. This is because as $L_0 \uparrow$, $f'_K(K_1, L_1) \uparrow$ and $f'_L(K_1, L_1) \downarrow$; making the condition hold. It does not change anything else (i.e. I_K remains the same).

The basic idea is that if the optimization problem is written in terms of choosing ratios (skills and investment), then increasing initial skills-stock does not change the two optimal allocation conditions. Hence the optimal ratios remain unchanged.

*The initial stock of skills, does not have any affect on the **proportion** of skills allocated to the production and proportion of investment allocated to physical capital.*

4.4 Optimal Ratios for Cobb-Douglas

If we assume the Cobb-Douglas functional form for production function, then the optimality conditions give -

⁶For CRS Case:

$$\frac{f'_L(K_0, L_0)}{f'_L(K_1, L_1)} = g\left(\frac{L_0}{L_1}\right) = g\left(\frac{L_0}{L_0 + L_0 * (1 - L_s) * \left(\frac{I - I_K}{C_L}\right)}\right) = g\left(\frac{1}{(1 + (1 - \left(\frac{L_s}{L_0}\right)) * \left(\frac{I - I_K}{C_L}\right))}\right)$$

$$\frac{\alpha}{(1-\alpha)} * \frac{L_1}{K_1} = \frac{(L_0 - L_s)}{C_L} \quad (5a)$$

$$\left(\frac{K_0}{L_0}\right)^{-\alpha} = \beta * \left(\frac{I_0 - I_K}{C_L}\right) * \left(\frac{K_1}{L_1}\right)^{-\alpha} \quad (5b)$$

These two equations give the optimal ratios for given parameter values.

Figure 1 shows the movement of these optimal ratios as initial capital stocks goes up. For the parameter values used, the results are consistent with the predictions ⁷. The ratio of skills used in production goes up and ratio of investment in physical capital seem to go down, as the initial capital stock rises.

Figure 2 plots the optimal ratios with total Investment moving up. The ratio of skills used in production remains the same (goes up very slightly), but ratio of investment in physical capital goes up with the increase in total investment endowment.

Figure 3 plots the optimal ratios with skills stock going up. Both ratios (skills used in production and investment in physical capital) does not change with the increase in L_0 .

Figure 4 plots the optimal ratios with α (capital share of output) going up. The ratio of skills used in production goes down and ratio of investment in physical capital goes up with the increase in parameter α for Cobb-Douglas production function.

5 Service Production Function

Technology is given by -

$$N_{capacity} = N^K(K) \text{ or } K_{threshold} = K^N(N) \quad (6)$$

These functions define,

⁷The fval(value of the equation evaluated at solution) in Matlab for some of the points were non-zero.

Figure 1: Optimal Ratios: Capital Stock Movements - Cobb Douglas

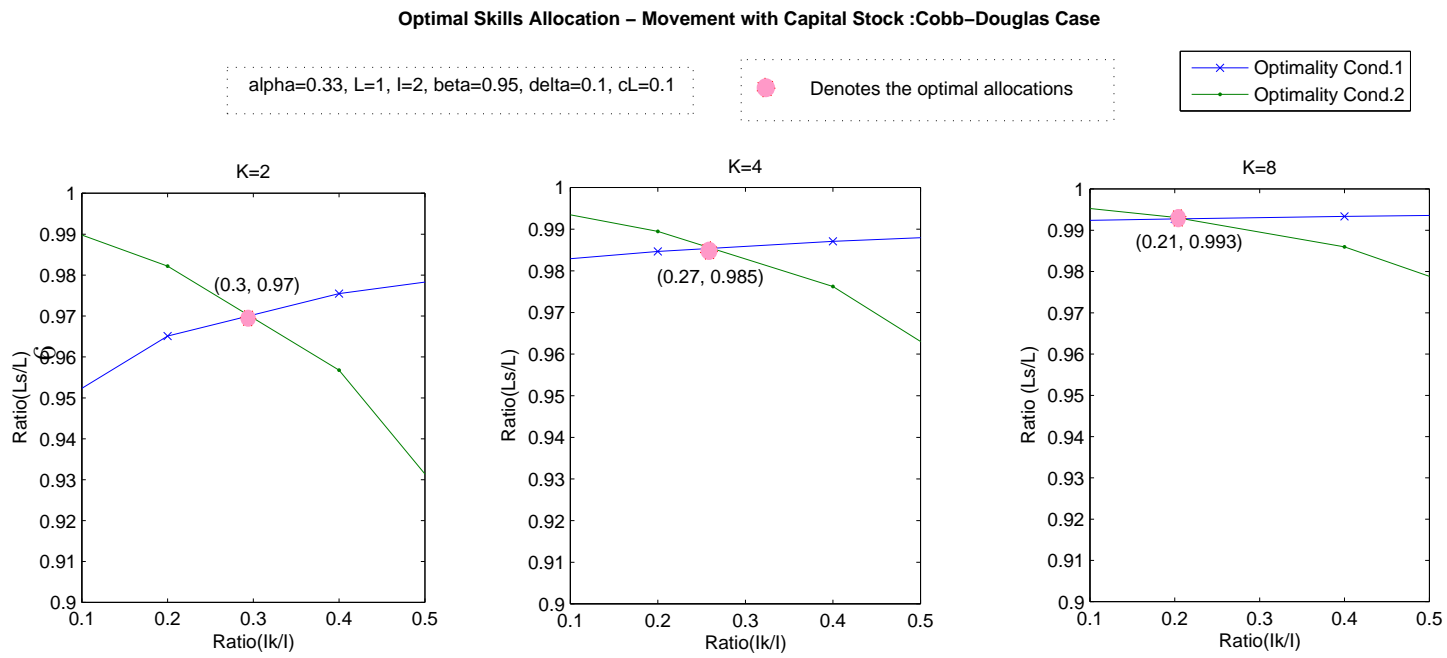
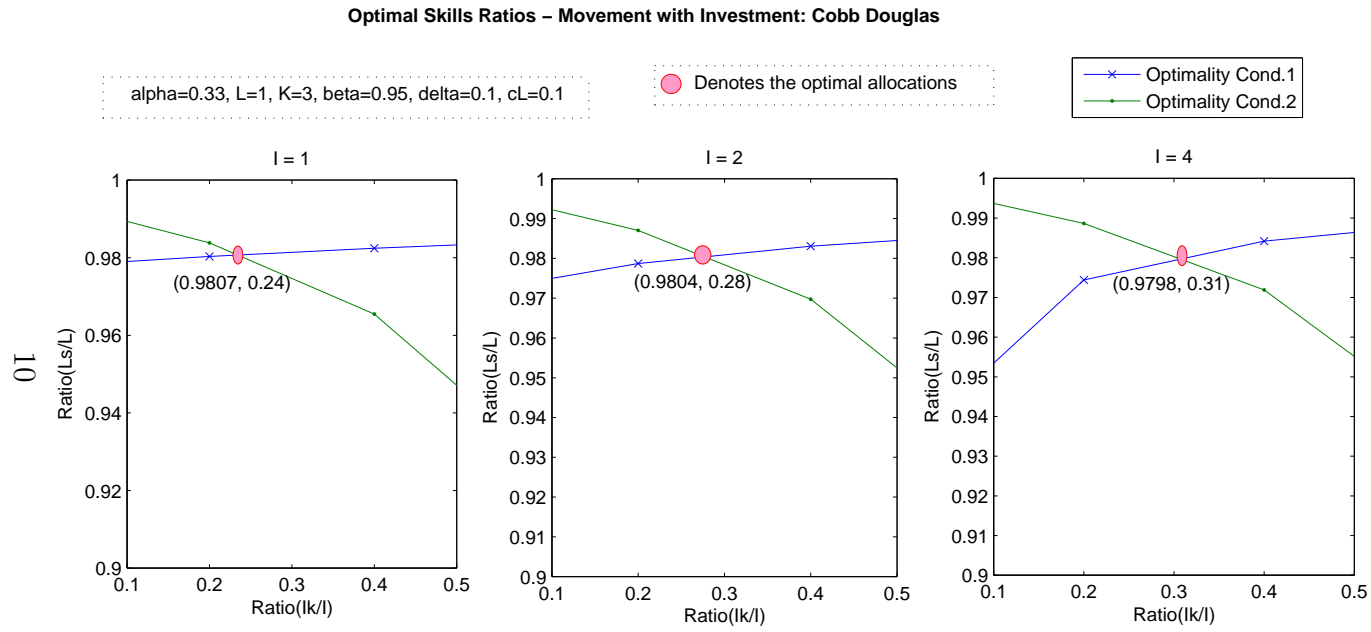


Figure 2: Optimal Ratios: Movements in Total Investment - Cobb Douglas

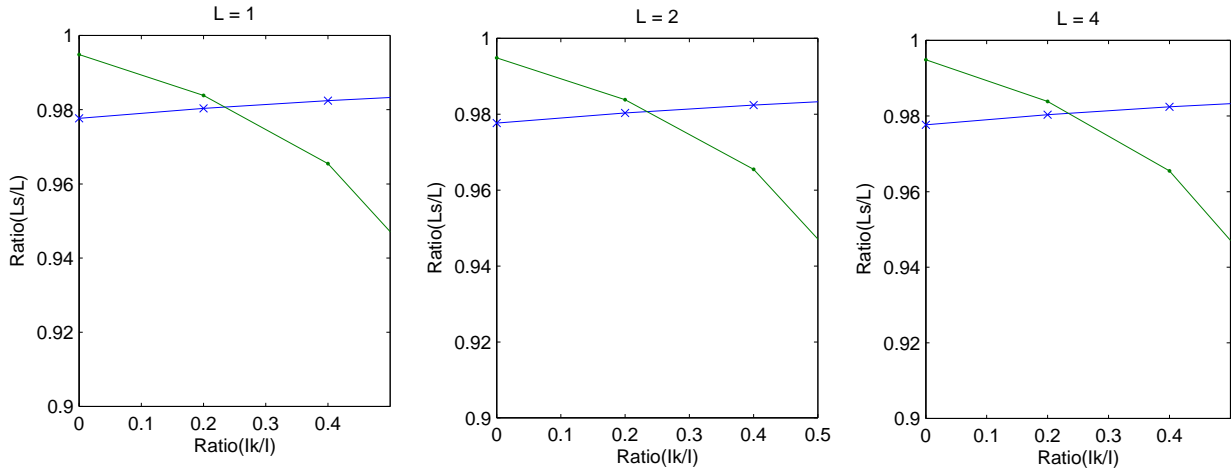


10

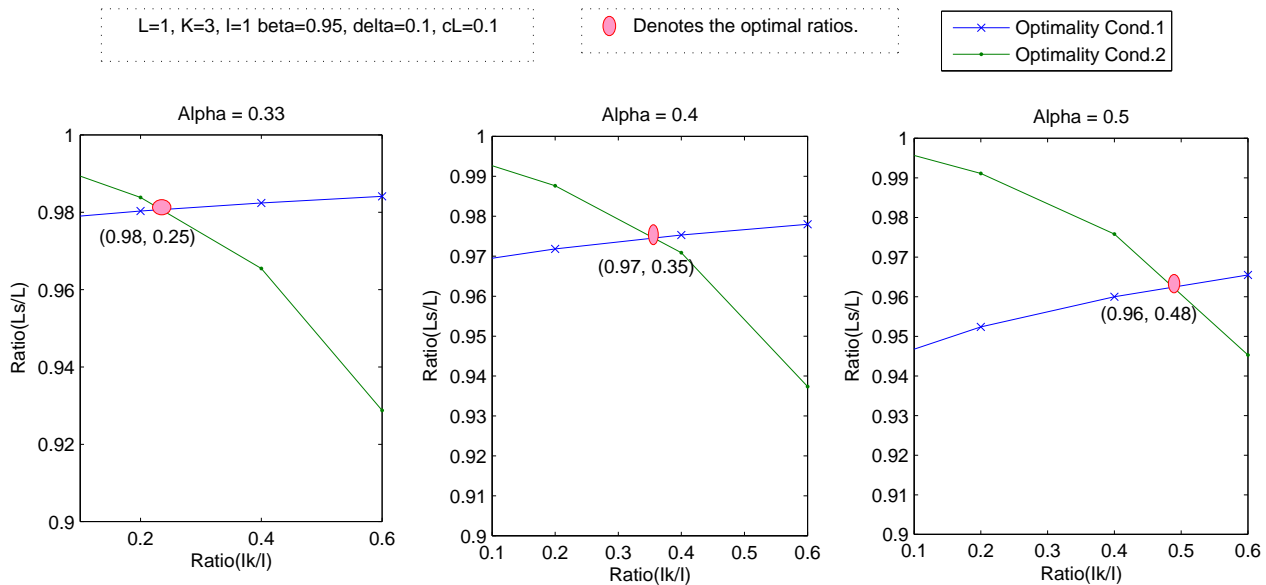
Optimal Skills Ratios – Movement with Skills Stock : Cobb – Douglas

alpha=0.33, K=3, l=1, beta=0.95, delta=0.1, cL=0.1

—x— Optimality Cond.1
—•— Optimality Cond.2



Optimal Skills Ratio – Movement with Capital Share of Output : Cobb – Douglas



- Number of maximum transactions that can be performed given hardware K .

- Minimum capital required to perform N transactions.

Similarly, there is technological limit (N_{max}, K_{max}) which is the maximum number of transactions possible and minimum hardware-capital required for that.

The production function is given by -

$$f(K_s, L_s) = A_s(K_s) * g_s(L_s) \quad (7a)$$

$$A_s = \begin{cases} N_{max}, & \text{if } K_s \geq K_{max} \\ N_s^K(K_s), & \text{otherwise} \end{cases} \quad (7b)$$

In the production function described above, $A_s(K_s)$ represents the software productivity which depends on hardware.

If linear functional form is assumed, then -

$$g_s(L_s) = L_s$$

$$N_s^K(K_s) = N_{max} * \frac{K_s}{K_{max}}$$

The role of innovation in production context is to make $N_{max} \uparrow$ or make $K_{max} \downarrow$. This will increase the total services production for a given amount of physical and skills capital.

6 Empirical Testing

Using dataset on industry-wise labor and capital allocations, following predictions can be tested:

1. Comparison based on capital stock:

In countries with higher capital stock -

- Higher portion of labor is employed in production.
- Higher portion of investments goes into training.
- Training sector is capital intensive. (More capital and less labor)

2. **Comparison based on investment:**

In boom-times (with higher investments) -

- Higher share of investment goes into physical capital.
- Lower share of labor is used in production.

3. These ratios are independent of total labor stock.

4. Movement of these ratios based on the capital share (α).