Skills Distribution, Migration and Wage Differences in Pure Service-Exchange Economy

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Abstract

This paper considers an economy with skilled agents exchanging their services. Using Cobb-Douglas preferences, the paper shows that there exists an optimal (average welfare maximizing) skills' distribution. This optimal distribution is independent of productivity and is welfare equalizing.

If the skill-distribution is not optimal, then some agents are better-off than others. In such a scenario, migration in some sectors is average-welfare improving while inviting skilled-agents in others reduces average welfare.

"Productivity increase of worse-off sector" without changing the overall skills' composition of economy increases the wage gap.

1 Introduction

In services context, most of the marginal production takes place using labor only. Once infrastructure is set up, it can be used for services' production using the required skills (subject to technological limit). The input to the production process are time and labor-skill. Hence for each time period the production only depend on the stock of skills in the economy. If we treat these skills as agents' endowments, then the economy can be represented by an exchange economy. This makes the model easier to solve and the results easier to interpret.

This kind of argument is more valid when it comes to **Consulting Services**, where a person's knowledge and skills are the most important (and often the only) inputs to the production process.

Some of the relevant questions are: why some skills (like Doctor and Lawyer) are valued more; is there an optimal skills' distribution; if government can control this distribution (by training and/or immigration), how should it make the decision; are there good and bad (not-so-good) skills/ sectors?

In most major economies fraction of Service sector in GDP is growing everywhere. **Specific** Skills are required for production of services (unlike capital which is mostly homogeneous). Usually, individual agents decide which skills to acquire, government has no control over it. But skills' composition is crucial for economy, government should take an aggressive approach to determine which skills its agents acquire (by investing/ providing subsidies etc.)

Technology (production and consumption) is of immense important in service sector. A better understanding of government technological policy in terms of **Formalizing** and **Implications** has huge potential.

Having optimal skills' composition translates into comparative advantage in today's Global-Service-Economy. But not much has been done to formalize the government technology policy as in: Which skills to promote? (via immigration or training) Which sectors/ technologies to promote and How?

This paper emphasizes the importance of skills' composition of economy, distributional aspects due to difference in skills of agents and presence of technological-externalities within a sector.

2 Basic Model Setup

Consider an economy with N agents skilled to perform various services. They get utility from consuming these services.

- Service production requires only the labor in form of agent's skill (and time). The infrastructure capital required to perform these services is already available.)
- There is a market for services exchange where agents trade in services with each-other. (Government provides coupons that help solve credibility problems).
- There are N services and each requires a unique skills to produce it.[1]
- Utility: $u(S)$. Defined over N services.
- Skills' Endowment Matrix: Ω.
  It is an NxN matrix of 0s and 1s and maps agents to the skills, indicating whether an agent has a particular skill or not.
- Productivity Matrix: A^{Ω}
  It is a NxN matrix and maps skills to services indicating how much of service (no. of transactions) can a particular **skill** can produce. It is a **DIAGONAL matrix** based on our setup. [2] [3]
  - Here it is assumed that one skill can produce more than one services. Alternatively, it can be assumed that one service requires more than one skills to produce it (Requirement matrix).
- Total Production: $Y = [1, 1, ...1] * \Omega * A^{\Omega}$
- Income vector: $I = \Omega * A^{\Omega} * P^{T}$, where P is the price vector $[p_1, p_2, ..., p_N]$.

The model has a nice feature that skills' distribution over the agents in the economy determines the supply of each service and thus the price of that service (**Because production requires only labor/ skills**). Another feature is that their income also depend on the price vector, which depends on skills' distribution and preferences.

Assumptions

[1] This is just a simplification. In general, one service may require more than one skills and similarly one skills can produce more than one services.

[2] Using this setup instead of having a direct NxN Agent-productivity matrix is more logical. Since productivity of a particular agent may be hidden information, while his skills are known by the certifications he holds and each skill-certification standard will have information about the service production.

[3] Another advantage is that Government can control these two parameters by asking the agents to take these professional-certifications (control Ω) and by changing the standard of these certifications(manage A^{Ω}). Similarly, **productivity shocks affect the whole industry rather than a particular person**.

1. $u^i(s_1^i, s_2^i, ...s_N^i) = \prod_j (s_j^i)^{\alpha_j}$

2. $\Omega_i = [0, 0, ...1, 0, ...0]$.
   Each agent has only one of the skills.

Solution:

Using optimal service demand result for Cobb-Douglas functional form, total demand for service i:

$\Sigma_j(\frac{\alpha_i * I_j}{p_i}) = \frac{\alpha_i}{p_i} * \Sigma_j(a_{jj} * p_j)$.

Total production of service i is a_i. Hence the market clearing price is given by -

$p_i = \frac{\alpha_i * \Sigma_j(a_{jj} * p_j)}{a_i} \Rightarrow \frac{p_i}{p_k} = \frac{\alpha_i / a_i}{\alpha_k / a_k}$

Price of a service is proportional to the ratio of preference parameter of that sector divided by the productivity of that sector.

The utility level of i^{th} agent is:

$u_i(S) = \Pi_j((\frac{\alpha_j * I_i}{p_j})^{\alpha_j}) = \frac{I_i}{\Sigma_k(a_{kk} * p_k)} * \Pi_j(a_j)^{\alpha_j}$

$\Rightarrow \frac{u_i}{u_k} = \frac{I_i}{I_k} = \frac{a_i * p_i}{a_k * p_k} = \frac{\alpha_j}{\alpha_k}$

Utility ratio of agents is equal to the ratio of importance of their skills (i.e. of the services their skills produce).
For example, doctors will be better off than workers if medical services has a higher α in the preferences.

This ratio does not depend on productivities of the sector.
Hence if their is a productivity shock in one of the sector, gains are shared by all the agents (sectors) in the ratio of their skills' importance.

3 Optimal number of skilled agents

For a consumer some services are more important than other. Hence it is better for an (closed) economy to produce more of the important service. Given consumer's preference the optimal ratio of services' production (and thus the skills/ labor required to produce that amount) can be determined.

This will help policy-makers in deciding which skills government should promote via immigration policy or tax/ other incentives.

Extending the above model for the case of 2 services, medical services (D) and construction services (W) which require Doctor's and construction Worker's skills.

- There are d Doctors and w Workers.
- Each type of agent needs some training (or certification exam) before he can provide the service. It can be thought of as **cost of acquiring skills**.
- Their productivity is given by, $A = \begin{matrix} 1 - t_D & 0 \\ 0 & 1 - t_W \end{matrix}$.

  This can be explained by assuming that t_D and t_W is the cost (the time required out of total endowment of 1 unit) for doctors and workers to get training/ certification in their profession. Hence they can now perform $(1 - t_D)$ and $(1 - t_W)$ transactions respectively.
- New agents can join the economy (i.e. government can invite immigrants from abroad or promote education so that previously unskilled people can take part certification exchange).

Market clearing condition for D becomes:

$d * (\frac{\alpha_D * p_D * (1-t_D)}{p_D}) + w * (\frac{\alpha_d * p_W (1-t_W)}{p_D}) = d * (1 - t_D)$

$\Rightarrow \frac{p_W}{p_D} = \frac{d * \alpha_W * (1-t_D)}{w * \alpha_D * (1-t_W)}$

The price becomes, $p_D = \frac{w * \alpha_D * (1-t_W)}{(w * \alpha_D * (1-t_W)) + (d * \alpha_W * (1-t_D))}$

Utility is given by, $[u_D = (\frac{w}{d})^{\alpha_W} * \alpha_D * (1 - t_D)^{\alpha_D} * (1 - t_W)^{\alpha_W}]$

and $[u_W = (\frac{d}{w})^{\alpha_D} * \alpha_W * (1 - t_D)^{\alpha_D} * (1 - t_W)^{\alpha_W}]$
The usual supply-demand comparative statistics holds.

- As $d \uparrow, p_D \downarrow$ and as $w \uparrow, p_D \uparrow$. **More doctors make price of medical services cheaper, while more workers make it costlier**.

- As $d \uparrow, u_D \downarrow$, but $u_W \uparrow$. **More doctors make existing doctors worse off while existing workers become better off**.
- As $w \uparrow, u_W \downarrow$, but $u_D \uparrow$. **More workers make existing workers worse off while existing doctors become better off**.
- As $t_D \downarrow$ or $t_W \downarrow$, $u_D \uparrow$ and $u_W \uparrow$. **Productivity gains in any sector or Reducing (any of) the training costs makes all consumers better off.**

3.1 Total Welfare

Since increase in d affects the welfare of Doctors and Workers in opposite way, it will be interesting to see what happens to the total welfare when a new doctor joins the economy (is born or immigrates).

Define total welfare, $U = w * u_W + d * u_D$

$\frac{\partial U}{\partial d} = \alpha_D * (\frac{w}{d})^{\alpha_W} * (1 - t_D)^{\alpha_D} * (1 - t_W)^{\alpha_W}$

and $\frac{\partial U}{\partial w} = \alpha_W * (\frac{d}{w})^{\alpha_D} * (1 - t_D)^{\alpha_D} * (1 - t_W)^{\alpha_W}$

- Both $\frac{\partial U}{\partial d}$ and $\frac{\partial U}{\partial w}$ are positive. Hence total welfare goes up if a new agent enters the economy irrespective of its skill endowment.
- To decide which skills should government promote (i.e. via immigration policy or investment in education etc.), one can compare the increase in total benefit by each skilled worker.
  - Doctors are more beneficial to the economy compared to the construction workers if $\frac{\partial U}{\partial d} > \frac{\partial U}{\partial w}$.
  - **Prefer doctor if** $\frac{d}{w} < \frac{\alpha_D}{\alpha_W}$
  - **Prefer worker if** $\frac{d}{w} > \frac{\alpha_D}{\alpha_W}$
  - **If** $(\frac{d}{w})^* = \frac{\alpha_D}{\alpha_W}$**, total welfare increase does not depend on the skill of agent joining the economy** (by birth, training or immigration).

3.2 Average Welfare

When a new agent joins the economy, the total endowment of the economy increases (and as a result total welfare). Therefore total welfare may not be the correct deciding factor for the government. Decision makers may want to see the effect of new skilled immigrant on average welfare.

Average welfare, $\overline{U} = \frac{w^{\alpha_W} * d^{\alpha_D} * (1-t_D)^{\alpha_D} * (1-t_W)^{\alpha_W}}{w+d}$

$\Rightarrow \frac{\partial \overline{U}}{\partial d} = \frac{(1-t_D)^{\alpha_D} * (1-t_W)^{\alpha_W} * [w^{1+\alpha_W} * \alpha_D * d^{-\alpha_W} - \alpha_W * w^{\alpha_W} * d^{\alpha_D}]}{(w+d)^2}$

$\frac{\partial \overline{U}}{\partial d} > 0$ will require $\frac{\alpha_D}{\alpha_W} > \frac{d}{w}$.

- **A new doctor increases the average welfare if $\frac{d}{w} < \frac{\alpha_D}{\alpha_W}$ and decreases the average welfare otherwise.**
- **A new construction worker increases the average welfare if $\frac{d}{w} > \frac{\alpha_D}{\alpha_W}$ and decreases the average welfare otherwise.**
- **If $(\frac{d}{w})^* = \frac{\alpha_D}{\alpha_W}$, then average welfare does not increase by change in the number of skilled agents($\frac{\partial \overline{U}}{\partial d} = \frac{\partial \overline{U}}{\partial w} = 0$).**

Social Planner's Optimal Skills Allocation:

If there is a social planner faced with problem of allocating the stock of total population N to one of the two sectors (Medical Services, D and Construction Services, W) to maximize the average welfare, the optimal solution would be -

$\max_d [\frac{(N-d)^{\alpha_W} * d^{\alpha_D} * (1-t_D)^{\alpha_D} * (1-t_W)^{\alpha_W}}{N}]$

This gives $(\frac{d}{w})^*_{Social-Planner} = \frac{\alpha_D}{\alpha_W}$, which is same as above optimal ratio.

3.3 Steady-State and Planned Expansion

If economy has skills distributed in this optimal ratio, expanding the economy (i.e. adding a new agent) will not change the average welfare. But if government decides to expand the economy (increase the size), then it should plan the expansion in a way such that this optimal ratio is maintained. This way expansion will not reduce the average welfare.

Optimal ratio is welfare-equalizing:

1. At this **ideal** $\frac{d}{w}$ ratio, the price of doctor's service becomes:

   $p_D^* = \frac{(1-t_W)}{(1-t_W)+(1-t_D)}$.

   $\Rightarrow \frac{p_D^*}{p_W^*} = \frac{(1-t_W)}{(1-t_D)} \Rightarrow Income_{worker} = Income_{doctor}$

2. **If economy has skilled agents in this ratio (ratio of αs in the preferences), then both kind of agents (doctors and construction workers) are equally well off**. (i.e. $u_D(\frac{d}{w})^* = u_W(\frac{d}{w})^*$)

3. **At optimal skills' ratio, the relative price of a service is inverse of the ratio of respective sectors' productivities.**

Figure 1 shows various expansion paths for an economy. It has 100 agents (in ratio 1:1, while the optimal is 3:2) to start with and wants to double its size. As shown in the figure, unplanned expansion (increasing only doctors or only workers) in fact reduces the average welfare.

Also notice that doctors end up at a lower utility level in the optimal expansion case. This highlights the presence of **lobbying** or resistance to expand from a particular sector. Currently well-off sector stands to lose from moving toward the optimal distribution.

3.4 Investment in Training Facilities

Even though the cost of acquiring the skills (certification or training) is welfare reducing, it is beneficial to compare the costs in different sectors. In particular, when is the case that:

$$\frac{\partial \overline{U}}{\partial t_D} < \frac{\partial \overline{U}}{\partial t_W} \Rightarrow \frac{(1-t_D)}{(1-t_W)} < \frac{\alpha_D}{\alpha_W}$$

- In making investment decision for the skills training sector, so that to reduce the cost of training or certification, **government should use the above relation to find out which cost affects average welfare more severely**.

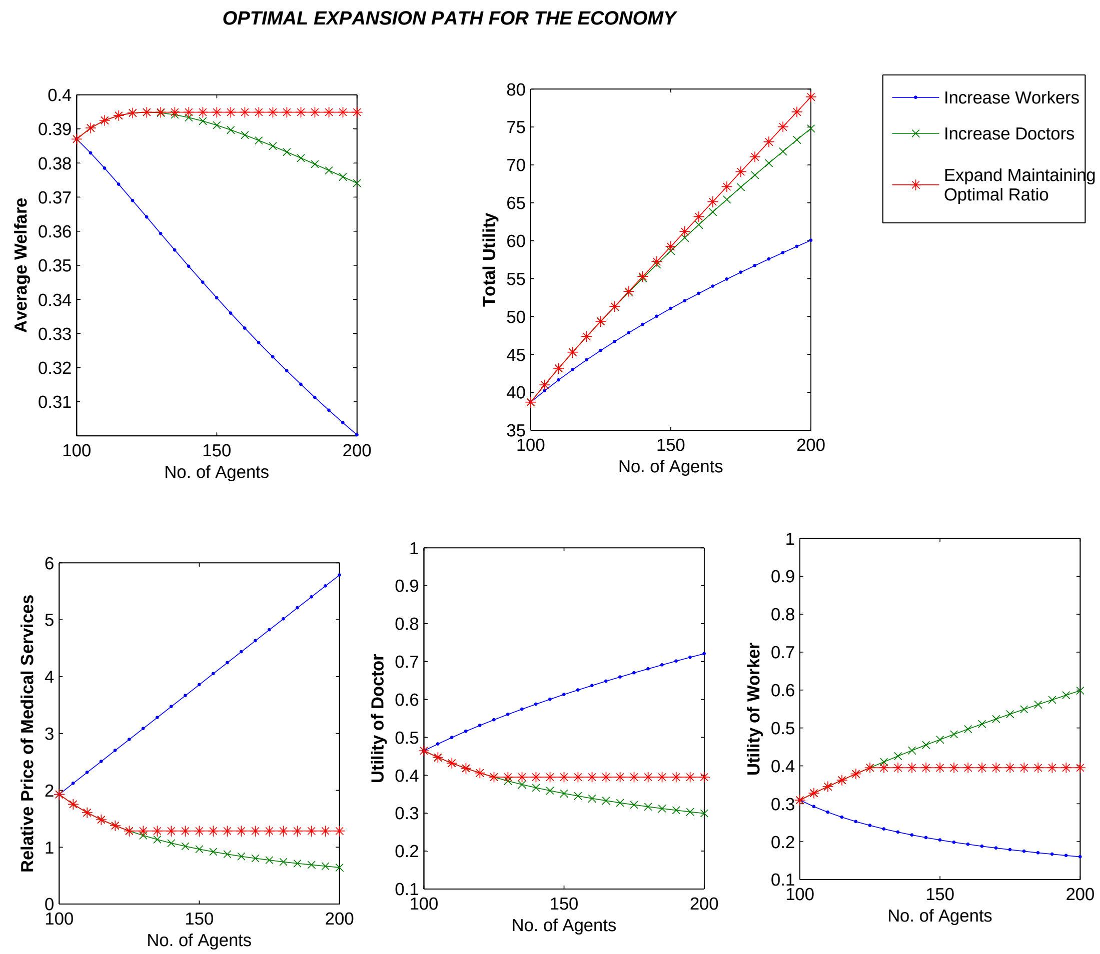

Figure 1: Expansion Path for Cobb-Douglas

3.5 Productivity Shock

Now we study the effect of productivity shock on optimal skills' distribution. We can introduce the productivity shock in one of the two ways.

1. Productivity of each agent increases to 2 i.e. now each doctor can produce $2 * (1 - t_D)$ medical services each period.

2. Cost goes down (Agent can complete his training in less time) i.e. now each doctor can produce $(1 - \frac{t_D}{2})$ medical services each period).

Solving for optimal skills' ratio gives the same result, i.e. $(\frac{d}{w})^* = \frac{\alpha_D}{\alpha_W}$. The intuition behind this result is that increased productivity leads to increased doctors' income, which in turn increases the demand for construction services. This changes the prices (i.e. $\frac{P_D}{P_W} \downarrow$). Optimal distribution of skills remain unchanged.

Also:

- A positive productivity shock in **ANY** sector increases the average welfare and individual utility for each type of agent (for any skill-distribution).

- The increase is **EQUAL** only if the skills' ratio is optimal.

4 Creative Destruction

- As economy develops, new services are invented and people start consuming these services. As a result, the share of income spent on old services goes down.

- Skilled labor should also be added (or new immigrants are brought in) to new services sector, so that this kind of optimal ratio is maintained.

- If new labor can not be added, then only way to achieve the optimal ratio is to move existing labor from one of the sector (whose income share has gone down) to this new sector.

- This seems to be a similar concept like creative destruction.

4.1 Effect of New Service Invention

Extending the 2 services model, now suppose that a new service Internet (denoted by I) is invented. After a while consumer form preferences over all three of the services.

For ease in calculation, also assume that $t_D = t_W = t_I = 0$.

Market clearing condition for medical services -

$d * \alpha_D + w * \alpha_D * \frac{p_W}{p_D} + i * \alpha_D * \frac{p_I}{p_D} = d \Rightarrow d * \frac{(1-\alpha_D)}{\alpha_D} = w * \frac{p_W}{p_D} + i * \frac{p_I}{p_D}$

Solving this with similar conditions for other two markets gives -

$$\frac{p_D}{p_I} = \frac{i * \alpha_D}{d * \alpha_I} \tag{1a}$$

$$\frac{p_W}{p_I} = \frac{i * \alpha_W}{w * \alpha_I} \tag{1b}$$

$$\frac{p_D}{p_W} = \frac{w * \alpha_D}{d * \alpha_W} \tag{1c}$$

The utility level of a doctor becomes -

$u(d) = \alpha_D * (\frac{w}{d})^{\alpha_W} * (\frac{i}{d})^{\alpha_I}$

Average utility is given by -

$\overline{U} = \frac{d^{\alpha_D} * w^{\alpha_W} * i^{\alpha_I}}{w+d+i}$

$\frac{\partial \overline{U}}{\partial d} \geq 0 \Rightarrow \frac{w+i}{d} \geq \frac{\alpha_W + \alpha_I}{\alpha_D}$
$\frac{\partial \overline{U}}{\partial w} \geq 0 \Rightarrow \frac{d+i}{w} \geq \frac{\alpha_D + \alpha_I}{\alpha_W}$
$\frac{\partial \overline{U}}{\partial i} \geq 0 \Rightarrow \frac{d+w}{i} \geq \frac{\alpha_D + \alpha_W}{\alpha_I}$

Proposition 1: There exists an optimal ratio of (d:w:i). [4]

Proof:
$\frac{\partial \overline{U}}{\partial i} = 0; \frac{\partial \overline{U}}{\partial w} = 0 \Rightarrow d = \frac{w * \alpha_D}{\alpha_W}$

[4] Optimal in the sense that at this ratio the average utility is at its maximum.

Similarly solving other sets of equation gives optimal ratio $(d : w : i) = (\alpha_D : \alpha_W : \alpha_i)$.

Proposition 2: All three of $\frac{\partial \overline{U}}{\partial d}, \frac{\partial \overline{U}}{\partial i}, \frac{\partial \overline{U}}{\partial w}$ can not be all positive.

Proof:
Assume $\frac{\partial \overline{U}}{\partial d} > 0; \frac{\partial \overline{U}}{\partial w} > 0$. These two imply:

$\frac{w+i}{d} > \frac{1-\alpha_D}{\alpha_D}; \frac{i+d}{w} > \frac{1-\alpha_W}{\alpha_W}$

Adding these two inequalities gives:

$\frac{(1-\alpha_i}{\alpha_i} > \frac{w+d}{i} \Rightarrow \frac{\partial \overline{U}}{\partial i} < 0$

Proposition 3: If skills' distribution is not at optimal level, some of the sectors will be better off than others (in terms of agents' welfare).

Proof:
From equations 1, it is evident if $(d : w : i) \neq (\alpha_D : \alpha_W : \alpha_I)$ then prices will be different. Hence agent's income (or welfare) in each sector will be different.

4.2 Feasibility and Rush for First-Mover's Advantage

If government ensures that after such a change in taste, it will bring in the scarce skills (either through immigration or through training of new agents) then there will not be any of the so called destruction. Sectors which have their income share go down will not be worse off, since at optimal allocation the utility is equal across all the sectors.

However this kind of policy may not be always feasible, due to immigration laws of country or due to lack of unskilled new labor which can be trained in the new sector. If that is the case agents in worse-off sector may decide to acquire the new skills themselves (supposing that they can do it without any cost). They have an incentive to do it sooner rather than later because the longer they wait the smaller the welfare gain gets (since other agents will be joining the new sector and the ratio moving more towards optimal ratio).

4.3 Simulation

Even though when it is not possible for government to get the new agents, it should control the training and movement of the labor to the new sector so that new optimal ratio can be reached.

If agents are allowed to decide it themselves it may take a lot more time to reach optimal and some the skill distribution may never achieve the steady state.

The situation is simulated as follows.

Consumers have preferences such that $(d : w : i)^* = (3 : 1 : 1)$. At the beginning, economy has 100 agents with 60 doctors, 35 workers and 5 Internet experts. If there is a government policy in place which strives for optimal skills' distribution, it will train 15 workers in Internet skills and move them to sector I. If we assume that each period only 5 people can be trained (say due to infrastructure limitation), then economy will reach optimal ratio in 3 periods.

Consider instead that 5 agents (who work in a non-highest-price sector) then they are trained in a sector with higher price than their current sector. If there are two such sectors, one is chosen at random. For example, at beginning of the period doctors will want to move to Internet sector, while workers may want to move to either Medical or Internet services sector.

As figures 2 and 3 show, if there is no policy in place the steady state may never occur and economy will experience a lot of fluctuations with agents moving from one-sector to other.

4.4 Growth Options

Above discussion shows that:

- If economy is already at an optimal skills-distribution, average welfare can not be increased by expanding one of the sector. In this case, productivity improvements will increase the average welfare (and individual welfare as well for all sectors).
- If economy is at the sub-optimal level, expanding all sectors of the economy is not going to be optimal. There will be at-least one sector expansion of which is going to decrease the average welfare.
- If a sub-optimal economy is not able to introduce new agents, some sector diminishes and other sector grows to increase the average welfare.

5 Wage Gap

Optimal ratio is welfare equalizing is a really neat result. The prevalent situation that workers are worse off than doctors (possibly because their

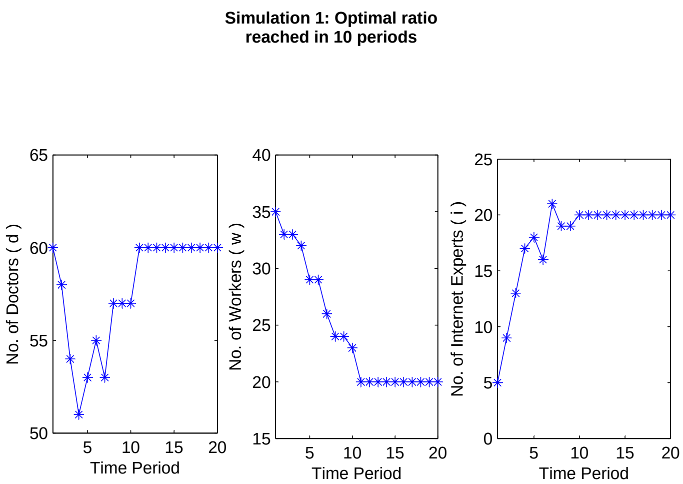

Figure 2: Simulation - Agents move to better sector, Steady state after 10 periods

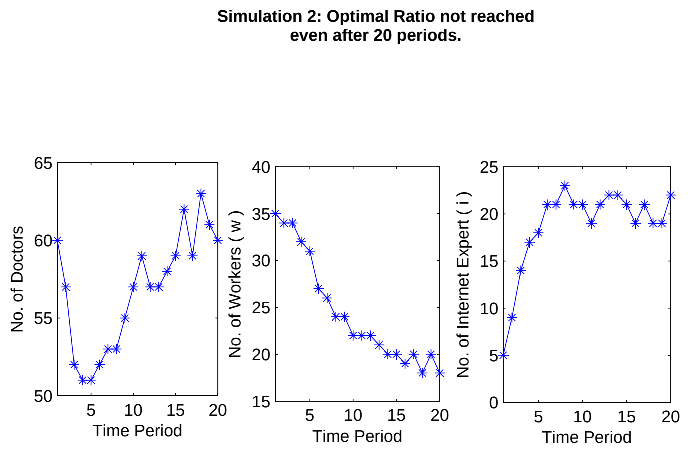

Figure 3: Simulation - Agents move to better sector, Steady state not reached in 20 periods

services are not valued much in the preferences) arises due to the non-optimal skills distribution.

From figure 1 it is clear that more well-off group has an incentive to lobby against the optimal distribution. Furthermore, better-off group gains more from increase in other sector's productivity. Hence following a "productivity increase of worse-off sector" approach only increases the welfare gap, as shown in the figure 5

6 Further Extensions

- Enriching existing person's skill set Ω rather than increasing the size of the economy will be a better way for the growth in average welfare.
- Finding which skills are more valuable (by preference structure and productivity) and skewing Ω to **move up in the value chain** in the global economy (OPEN economy extension of the model).
- Diversification (to insure against productivity shocks and taste-changes) vs. Specialization (patenting a particular high-value sector technology with WTO with lots of Enhancing complements).
- Use alternative setup for production technology (Requirement matrix rather than Productivity matrix).

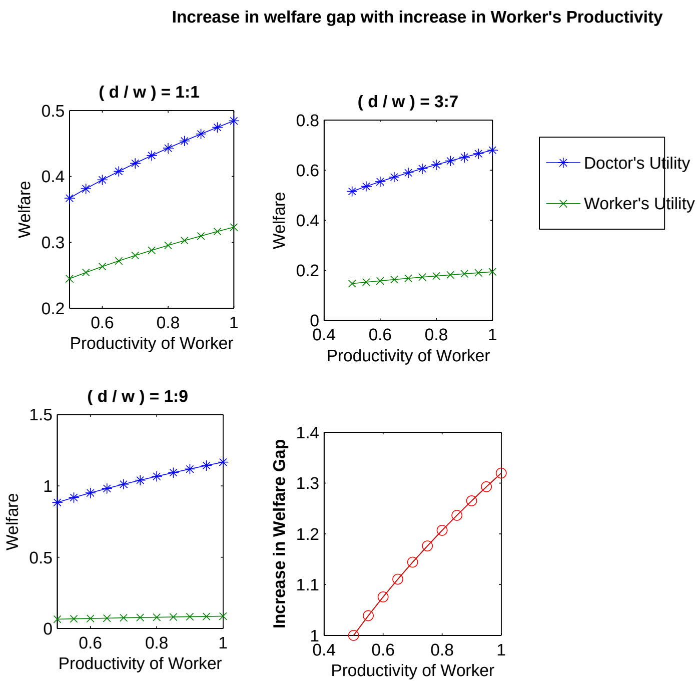

Figure 4: Increase in Welfare Gap with increase in Worker's productivity