

# Technological Progress on Consumption side: Consolidation and Prevalence of Complements

Abhay Gupta  
abhayg@interchange.ubc.ca

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## Abstract

This paper discusses the effect of technological innovation on consumption side. Apart from Quality effect (improvement in the quality of service or reduction in constant-quality price), there is another "**Consolidation effect**".

This takes form of more features (which can be enjoyed together) being included in the same service. This effect is driven by the Time-constraint in form of technological limit on consumption per unit of time. The effect is stronger if features being bundled together are complementary to each other.

Another aspect is shown by offering an alternative explanation of Engel's Law. If complementarity of a sector is affected by technological externalities, the income share spent on that sector changes. The direction of movement depends whether the tech progress has developed "enhancing" (positive) or "impeding" (negative) complements. Service sector has more of these enhancing complements and hence income share spent on service sector has gone up.

# 1 Introduction

It is a universally accepted fact that most important technological innovations of recent years are computer and Internet. The interesting question is that: "Were these innovations just some random events or were these a result of rational scientific evolution process?"

Even if we believe that probability of a successful scientific innovation is random, there is some decision involved in the choices we make when we decide which type of scientific research to undertake. There have been cases of accidental inventions of something unrelated to the desired objectives, but that should not be treated as the norm.

In fact, simple logic suggests that since we see research projects with specific objectives being funded everywhere, there should be a positive correlation between amount of resources dedicated to the research and the probability of its success. (The only scientific research in no-correlation scenario will be funded by the scientists themselves who get utility from doing research).

If we (or government or firms with shareholders as owners) make decisions on what kind of research should be funded, should not the choice of research projects (and in turn the actual scientific innovations) be guided by welfare maximization objective in mind.

Hence, the idea that these technological advancements of recent years do represent some economic optimization behind them seems worth examining.

In economics, technology is discussed mostly with regards to production. Consumer is affected by the technology via expansion in his choice and budget set (resulting in increased labor productivity and decrease in price of constant quality services).

But there is another channel via which innovation affects consumer. Technological innovations also expand the Time-budget set. It improves the life-span (which is now being used as a measure of welfare just like real income), but more subtly it reduces the time-cost of consumption.

This channel is more apparent in the services industry. In recent times, we have seen following trends -

- Transaction times are going down (rise in business process automation, online shopping etc.).
- Popularity of search engines.
- Consolidation of services (same provider offering many services in a bundle etc.).

I try to explain the idea of this channel is by constructing a simple model of service industry and studying the effect of a technological innovation in the sector.

Another major trend of past few decades is the rise in income share spent on services. There have been explanations using hierarchy of demands or subsistence concepts. But recognition that technological innovation plays a more important role in Service sector is missing. Many of the *new* services are developed with objective to increase the benefits we get from existing services rather than competing with the existing ones. For example wireless internet in coffee shop is supposed to increase the utility we get from enjoying the hospitality service of cafe.

In effect, such a technological progress acts as a positive externality for the service sector. This results in increased marginal utility from the service sector at the existing allocation. Consumer's re-optimization to equate the marginal utility per dollar across sectors guides him to consume more of the services. Hence the income share spent on service sector rises. This kind of acts as a **feedback loop** for the sector.

I explain this using a simple setup for Cobb-Douglas preferences and a sector specific **complementarity** parameter. If this parameter is enhancing (i.e. more services consumer uses, more utility he gets from existing ones), then income share spent on other sector goes down.

## 2 Why do lottery-winners take early retirements?

If budget is the only constraint faced by the consumer, we should not observe lottery winners and rich people taking early retirement. They should keep on working as long as their wages is higher than disutility from work. But in most of the cases people stop working after winning the lottery.

The reason is that instantaneous consumption is not possible (especially in services context). People need time to consume goods and services. Because of this consumer faces a *time-budget-constraint* along with the usual budget constraint. Generally, only the budget constraint is binding and time-constraint is non-binding.

But as income goes up, this time-constraint starts mattering.<sup>1</sup> Once time-constraint becomes binding, consumer chooses the optimal allocations of time on services equating utils obtained per unit of time (just like he decides to spend his income on services equating utils per unit of dollar).

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<sup>1</sup>See Metcalfe, 2001 [?] for more on innovation and time.

## 2.1 Technology in Consumption Context

It is true that most obvious way the technology affects a consumer is via expansion in available goods and services. But unlike many models where technology directly affects only the production side; technology plays very important in consumer context as well. This is due to presence of two somewhat subtle concepts.

### 1. Usage-time-cost :

It is apparent that to consume a service one needs to spend time. Hence each service has a time-cost (amount of time required to use or consume that service). This time-cost affects consumer's decision making since the total amount of time consumer has is limited.

### 2. Transaction-disutility :

Each service consumed gives some disutility as well. It is due to the inconvenience caused during *search*, *negotiation* and *transaction* process. This is specially important in the services context because the utility of a service consumption is the difference between start-stage-utility and end-stage-utility. Hence the process or transaction-disutility must be subtracted when calculating the welfare.

### Justification for including usage-time-cost and transaction-disutility:

Human and technological limitations restrict the amount of services a person can consume per unit of time. With technological advancements, this limit expands and consumption per unit of time increases.

Examples of Transaction Disutility -

1. Have to memorize all the login-passwords
2. Carry membership cards.

These transaction-disutility does affect consumer's decision making. That is why Internet services (e.g. online shopping, social-networking sites etc.) are becoming popular.

Many of the recent advancements in technologies are aimed at saving this transaction-disutility and usage-time-cost. For Example:

1. Mobile Phone, Media player, Camera and PDA functionalities combined.
2. Online Aggregation i.e can see all the bills by single login.

Another example is where these concepts show their importance is Newspaper. It is disappearing because its time-cost of consumption is higher than online news (where one can search and browse through relevant sections much faster).

### 3 Basic Model Setup

The model suggested here is similar in idea with Cheung and Ng, 2004 [?].

Consumer derives utility from various features which are offered in different services. In this context, state of technology determines:

- M - No. of consumable features.
- Y - (MxN) matrix, denoting the composition of services produced in the economy.
- T - Time required to consume the services <sup>2</sup>
- t - Transaction disutility from searching-negotiating-transacting for the services <sup>2</sup>

Basically, M is the dimension of *Feature Space* and Y is the mapping from *Service Space* to this feature space. T and t are functions defined on the service space.

#### Consumer Problem

A consumer solves the following optimization problem while choosing the service consumption vector S -

$$\max_S [u(F) - t(S)]; \tag{1}$$

subject to

$$F = Y.S \tag{2a}$$

$$P^S . S \leq I \tag{2b}$$

$$T(S) + T_{worked} \leq T_{total} \tag{2c}$$

$$I = w * T_{worked} \tag{2d}$$

#### 3.1 Features

The utility function u(F) is -

- Concave in each of its argument (  $u' \geq 0; u'' \leq 0$  )
- Non-decreasing in dim(F) (  $\frac{\partial u}{\partial M} \geq 0$  )

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<sup>2</sup>In general, it is a sum of N elements. Each element being a function of amount of the corresponding service.

Service usage-time-cost can not be negative.  $T(S) \geq 0$

Transaction-disutility is also assumed to be non-negative.<sup>3</sup>  $t(S) \geq 0$

For simplicity, I *assume* that -

$T(S)$  and  $t(S)$  are linear functions<sup>4</sup> and have same value for each of the service offered; i.e.

$$T(S) = T * (1_N)^T . S \text{ and } t(S) = t * (1_N)^T . S$$

(where  $T$  and  $t$  are constants determined by the technology).

### 3.2 Technology

In general, technology has three channels via it affects the welfare.

1. **Efficiency channel:** Decrease in parameters  $T$  and  $t$ .
2. **Consolidation Channel:** Offering more of existing features together bundled as one service.
3. **Expansion Channel:** Finding new features.

Consolidation and Expansion channels work through changes in the feature-composition,  $Y$  of services offered in the economy.

*Moreover, these two channels are more effective when the features offered in a service are complements to each other.*

#### **Proof:**

The lagrangian for optimization problem can be written as -

$$L = (u(Y.S) - t(s)) - \lambda * [w * T_T - w * T(S) - P.S]$$

First order necessary conditions imply that at least one of the following should hold -

$$u'.Y = t' \tag{3a}$$

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<sup>3</sup>This can be justified even for the services where one “enjoys” the search-negotiation-transaction process by adding this extra enjoyment as a feature of that service

<sup>4</sup>Taking linear transaction cost rather than fixed cost seems more inclusive. For example using telephone communication services requires a person to search/retrieve-dial the phone number each time he uses the service. With technical advancements this transaction cost goes down, since he no longer has to maintain a phone-book or manually search or dial the phone number. Voice dialing only requires him to say the name of the person he is calling.

$$P.S = w * [T - T(S)] \quad (3b)$$

Equation 3a is the usual *optimality condition* equating the MB of each service to its MC at optimal allocation, while other equation is the combined budget-constraint obtained by substituting for income.

### 1. Efficiency channel:

In case of linear transaction disutility,  $t' = t$ .

As  $t \downarrow$ , from equation 3a,  $u' \downarrow \Rightarrow F^* \uparrow$ .

Hence a decrease in transaction-disutility parameter is welfare increasing. This validates the presence of efficiency channel.

### 2. Consolidation channel:

Suppose that  $i^{th}$  feature is included in few services <sup>5</sup>. Then condition 3a implies that -

$$\left(\frac{\partial u}{\partial f_i}\right) * [\sum_j y_i^j] = t$$

If consolidation happens and it is offered in more services, then  $\sum_j y_i^j \uparrow$  and as a result  $\frac{\partial u}{\partial f_i} \downarrow \Rightarrow f_i^* \uparrow$ .

Hence offering more features in one service is welfare increasing. This shows how the consolidation channel works.

#### Note:

If this feature is substitute to other features, then there may be secondary effects in other direction. As  $f_i \uparrow, f_j \downarrow$ . Hence the total welfare change is usually not as large as in the case when this feature is complementary to lots of other features. Then the above channel will generate positive secondary effects ( $f_i \uparrow \Rightarrow f_j \uparrow$ ) generating even more welfare.

### 3. Expansion channel:

Suppose, feature i was not available earlier (i.e.  $\sum_j y_i^j = 0$ ). As technology expands its offered in some services <sup>5</sup>. Using optimality condition 3a, it follows that  $\left(\frac{\partial u}{\partial f_i}\right) \downarrow$ , causing a welfare increase. But if this new feature is complementary to existing ones, this inclusion will have secondary effect of  $f_j \uparrow$ , when j is complementary feature of i.

Hence finding new features to offer makes consumer better off. This shows the presence of expansion channel.

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<sup>5</sup>It is assumed that this marginal inclusion has negligible effect on price vector.

□

There are numerous examples showing these effects at work.

- iPod: The time-cost of this new activity is zero, since it can be used while doing other activities like running, travelling etc.
- Air Lines offering in-flight entertainment and Internet access; Restaurant offering live-music; Enjoying ADT security, air conditioning while sleeping: These are the examples of offering more features that can be enjoyed together.

As mentioned earlier, print-media is losing to Internet and TV. But why is that radios are still surviving? This is due to their complementarity with car driving. Another fact that in train/ public transportation newspaper is still being used also shows that time-cost and complementarity do play a role in consumer's decision making (since then newspaper is being enjoyed in the same time-cost as that of commuting as a complementary feature).

#### **Ideal Case:**

In an economy with no technological restrictions, consumers will be directly choosing the features. All the features will be offered in one service (to minimize usage-cost and transaction-disutility). Central planner will offer this service with features' compositions representing their optimal allocations.

$$Y = [(f_1)^*, (f_2)^*, \dots, (f_M)^*]$$

For Example: Supermarkets selling everything. Ubiquitous Internet offering all the services (entertainment, communication, education etc).

#### **A Note on General Purpose Technologies:**

This setup can be used to explain what is described as GPT in literatures and some examples include steam engine, electricity, computers, Internet etc. These are the technologies that are complementary to lots of other features or on which many other features can be added-on.

*Between 2 substitutes, one that is offered with lots of complementary features becomes successful.*

## 4 Aggregation of Services Results

**Lemma 1.** *It is better<sup>6</sup> to offer complementary features as a combined service.*

**Proof:**

Suppose  $M=2$  (i.e. feature space consists of only two elements) and they are perfect complements (preferences are represented by Leontiff function). The price of feature2 in terms of feature1 is  $p$ . Also assume that total available time is 1 unit and wage rate is also unity. Another assumption is that the price of combined service follows equation ??<sup>7</sup>.

a) With No Time Constraint -

|   |  |
|---|--|
| Case1: Offered as Combined Service ( <b>1S</b> )<br>$Y=[1,1]$   | Case2: Offered as Separate Services ( <b>2S</b> )<br>$Y=\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$  |
| Optimization Problem:<br>$\max_s [\min(f_1, f_2) - t * s]$<br>s.t.<br>$(1 + p) * s \leq I$<br>$[f_1, f_2] = [s, s]$ | Optimization Problem:<br>$\max_{s_1, s_2} [\min(f_1, f_2) - t * (s_1 + s_2)]$<br>s.t.<br>$s_1 + p * s_2 \leq I$<br>$[f_1, f_2] = [s_1, s_2]$ |
| Solution:<br>$u^* = \frac{(1-t)I}{(1+p)}$   | Solution:<br>$u^* = \frac{(1-2t)I}{(1+p)}$   |

Hence the optimal utility is higher when features are offered in form of a combined service (1S) rather than separate services (2S). The result is being driven by the fact that even though in 1S consumption is forced to be on  $f_1 = f_2$  line, it does not add any extra restriction since it coincides with the optimal allocation condition. Since in 2S there is extra transaction disutility,

<sup>6</sup>This result is about the **direction of future services innovation** and not about choosing a better one from existing services. Because if one service offers more utility than the other one, then the lower-utility service will disappear from the market.

<sup>7</sup>Since combining services may be welfare improving, companies may charge a premium. But in a perfectly competitive economy, the prices should follow implicit-feature-pricing condition (if combining the services does not increase the marginal cost)

hence it ends up at a lower optimal utility level.

b) Time Constraint -

|   |   |
|---|---|
| Case1: Offered as Combined Service ( <b>1S</b> )                                | Case2: Offered as Separate Services ( <b>2S</b> )   |
| Same as above with extra constraints -<br>$s * T + T_w \leq 1$<br>$I = T_w * 1$ | Same as above with extra constraints -<br>$(s_1 + s_2) * T + T_w \leq 1$<br>$I = T_w * 1$ |
| Solution:<br>$u^* = \frac{(1-t)}{(1+p+T)}$                                      | Solution:<br>$u^* = \frac{(1-2t)}{(1+p+2T)}$  |

The reasoning is similar as in earlier case. There is no loss by restricting consumption ratio in 1S, but it saves both the transaction cost and the consumption time-cost (i.e. usage cost).  $\square$

**Lemma 2.** *Offering substitute features in a combined service makes sense only when transaction-disutility or time-cost is high.*

**Proof:**

Suppose  $M=2$  (i.e. feature space consists of only two elements) and they are perfect substitutes (preferences are linear). The price of feature2 in terms of feature1 is  $p$ . Also assume that total available time is 1 unit and wage rate is also unity.

a) With No Time Constraint -

|  |   |
|--|---|
| Case1: Offered as Combined Service <b>(1S)</b><br>$Y=[1,1]$  | Case2: Offered as Separate Services <b>(2S)</b><br>$Y=\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$                                       |
| Optimization Problem:<br>$\max_s [(f_1 + f_2) - t * s]$<br>s.t.<br>$(1 + p) * s \leq I$<br>$[f_1, f_2] = [s, s]$ | Optimization Problem:<br>$\max_{s_1, s_2} [(f_1 + f_2) - t * (s_1 + s_2)]$<br>s.t.<br>$s_1 + p * s_2 \leq I$<br>$[f_1, f_2] = [s_1, s_2]$ |
| Solution:<br>$u^* = \frac{(2-t)*I}{(1+p)}$   | Solution:<br>$u^* = \begin{cases} (1-t) * I; & \text{for } p \geq 1 \\ \frac{(1-t)*I}{p}; & \text{for } p < 1. \end{cases}$               |

b) Time Constraint -

|   |   |
|---|---|
| Case1: Offered as Combined Service <b>(1S)</b>                                  | Case2: Offered as Separate Services <b>(2S)</b>   |
| Same as above with extra constraints -<br>$s * T + T_w \leq 1$<br>$I = T_w * 1$ | Same as above with extra constraints -<br>$(s_1 + s_2) * T + T_w \leq 1$<br>$I = T_w * 1$   |
| Solution:<br>$u^* = \frac{(2-t)}{(1+p+T)}$                                      | Solution:<br>$u^* = \begin{cases} \frac{(1-t)}{(1+T)}; & \text{for } p \geq 1 \\ \frac{(1-t)}{(p+T)}; & \text{for } p < 1. \end{cases}$ |

These optimal utilities are plotted using Matlab for different parameter values of t and T.

Figure 1 compares the optimal utility level of 1S and 2S cases without any time-cost(T). It shows that separate services (2S) case attain a higher optimal utility level, since it does not restrict the allocation on  $f_1 = f_2$  line. But as t (i.e. transaction-disutility) goes up, consolidation of services gives

higher utility (for a price range).

Figure 2 introduces the time-cost( $T$ ). It again shows that with perfect-substitute, features' consolidation gives higher utility only when  $T$  is very large (for a given  $t$ ).

□

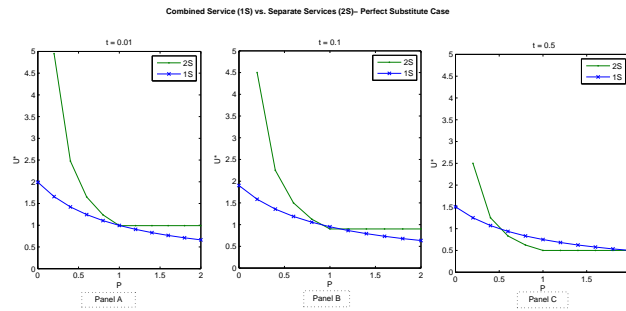


Figure 1: 1S vs. 2S Perfect Substitute case

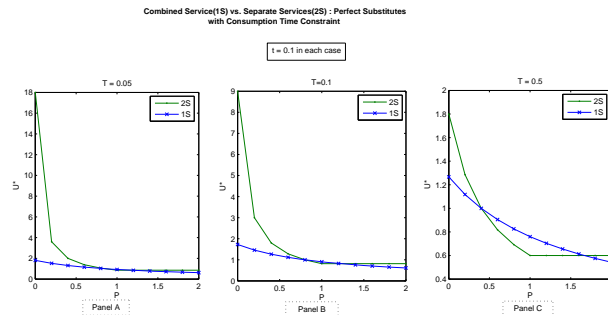


Figure 2: 1S vs. 2S Perfect Substitute case - With Time-cost

## 4.1 Other Preferences

### Cobb-Douglas:

Till now, I have considered two cases namely L-shaped indifference curves (perfect complements) and Linear indifference curves (perfect substitutes). Applying the above setup to preferences represented by indifference curves

between these two extremes where the utility function is given by Cobb-Douglas function -

$$u(f_1, f_2) = (f_1)^\alpha * (f_2)^{1-\alpha} - t.S$$

OPTIMAL SERVICE COMPOSITION -

Before comparing the consolidated vs. separate services, it is important to realize that choice of Y (available features in each service) is crucial. It restricts the allocation to be along a particular plane (a specific straight line in 2 features case). So, if consumer is allowed to pick up the composition of the service what will his optimal choice be?

Let  $Y = [1, \beta]$  be the service offering. The consumer's optimization problem is -

$$\begin{aligned} & \max_{s, \beta} [s^\alpha * (s\beta)^{1-\alpha} - t * s] \\ & \text{subject to } (1 + \beta * p) * s \leq I \end{aligned}$$

The optimal  $\beta$  will satisfy -

$$\beta^{-\alpha} * [(1 - \alpha) - (\alpha * \beta * p)] = p * t$$

For no-transaction-disutility case, i.e. for  $t = 0$ ; it gives -

$$\beta^* = \frac{(1-\alpha)}{\alpha * p}$$

This is the usual Cobb-Douglas solution, where consumer spends fixed proportion of his income on each feature.

*"Is consumer free to choose the service composition?"*

The answer is Yes in today's economy. In fact, most of the service being offered as flexible packages, where consumer can choose how many features does he want in the service bundle. Example: Cable channels; Mobile-phone services etc.

Figure 3 compares the optimal utility level for three cases -

- A) 1S:  $Y=[1 \ 1]$  and  $u^* = \frac{(1-t)}{(1+p)}$   
 B) 1S\*:  $Y=[1 \ \frac{(1-\alpha)}{\alpha * p}]$  and  $u^* = [\frac{\alpha^\alpha * (1-\alpha)^{1-\alpha}}{(1+p)} - t * \alpha]$

$$\text{C) 2S: } Y = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \text{ and } \begin{cases} u^* = (s_1)^\alpha * (s_2)^{1-\alpha} - t * (s_1 + s_2) \\ s_2 = \frac{(1-s_1)}{p} \\ s_1 - \alpha + t * (p-1) * p^{-\alpha} * (s_1)^{1-\alpha} * (1-s_1)^\alpha = 0 \end{cases}$$

Panel A shows that since 1S restricts the consumption to be on  $f_1 = f_2$  path, it is better than 2S only for a price range even though it saves transaction disutility of the second service.

Comparing Panel A and B shows that as  $t \uparrow$ , consolidation is better for whole price range.

From Panel A and C, it seems that 1S starts to dominate 2S for a broader price range as  $\alpha \uparrow$

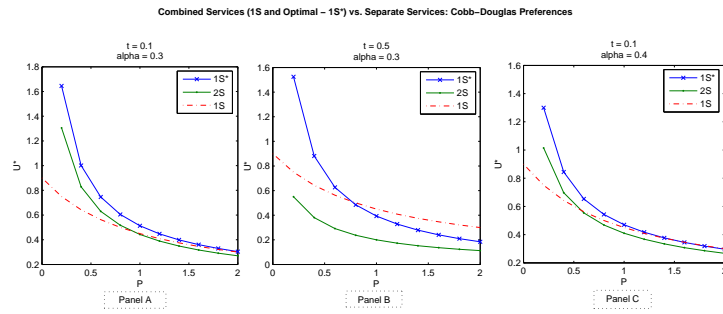


Figure 3: 1S vs.  $1s^*$  vs. 2S - Cobb Douglas Preferences

### CES Preferences:

The problem with Cobb-Douglas preferences is that income elasticity of consumption is always 1 which is in contrast with Engel's law. Another widely use functional form is CES preferences -

$$u(f_1, f_2) = [(f_1)^\rho + (f_2)^\rho]^{\frac{1}{\rho}}$$

If consumer is free to choose the composition of combined service, the optimal  $Y$  would be -

$$Y = [1 \ p^{\frac{1}{\rho-1}}]$$

The comparison of three utility levels is shown in Figure 4

A) 1S:  $Y=[1 \ 1]$  and  $u^* = [(\frac{1}{1+p}) * (2^{\frac{1}{\rho}} - t)]$   
B) 1S\*:  $Y = [1 \ p^{\frac{1}{\rho-1}}]$  and  $u^* = [(\frac{1}{1+p^{\frac{\rho}{\rho-1}}}) * (2^{\frac{1}{\rho}} - t)]$   
C) 2S:  $Y = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$  and  $\begin{cases} u^* = [(s_1)^\rho + (s_2)^\rho]^{\frac{1}{\rho}} - t * (s_1 + s_2) \\ s_2 = \frac{(1-s_1)}{p} \\ [(s_1)^\rho + (\frac{1-s_1}{p})^\rho]^{\frac{1-\rho}{\rho}} * [(s_1)^{\rho-1} - \frac{1}{p} * (\frac{1-s_1}{p})^{\rho-1}] + t * \frac{(1-p)}{p} = 0 \end{cases}$

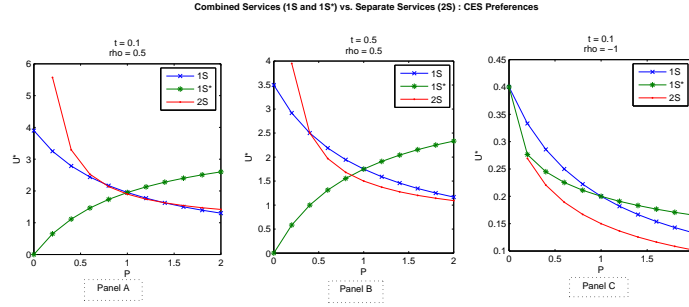


Figure 4: 1S vs. 1s\* vs. 2S - CES Preferences

The results for CES preferences are similar.

Consolidation of features becomes important as transaction parameter increases  $t \uparrow$  (comparing figures in panel A and panel B).

As the elasticity of substitution goes down, i.e.  $\rho \downarrow$ , the consolidation becomes important (as shown by 1S and 1S\* becoming better than 2S). Hence benefits from combining the features in one service is more if they are complements rather than substitutes.

## 5 Increase in income share spent on services

The most obvious explanation is that people's preferences have some unobservable environment variables. These environment variables change with time as new related technology and services are invented.

### Two types of Complements

1. **Impeding Complements:** Usual definition. One can not enjoy any of them unless all of them are consumed in a fixed ratio.

Example: Flash Light and Battery; Digital Camera and Memory. Construction Service: Roof and Walls. Medical Service: Orthopedic Surgeon and X-ray.

2. **Enhancing Complements:** Some services have the property that they give more pleasure when consumed together. Hence the functional form should be defined as something that **ADDS** to the enjoyment. In particular,  $u(X) \uparrow$  as  $N \uparrow$  where  $N$  is the dimension of  $X$  (number of complementary services being enjoyed together).

Example: Good Health is a enhancing complement to almost all other services. Computer and Telephone line. Construction: Heating and House. Medical: Physical Health and Mental Health.

## 5.1 Simple Functional Form

Let  $X$  and  $Y$  denote the agriculture and service sector respectively. Service sector  $Y$  offers  $N$  related services.

The sub-utility from this sector is  $C(y) * y$ , where  $C$  is the **complementarity function of the sector** and  $Y$  is the services consumed. This  $Y$  is represented in terms of individual services as  $[y_1^{\alpha_1^y} * \dots * y_N^{\alpha_N^y}]$

Complementarity function  $C(y)$  denotes the sector-wise interdependence or externalities. If these services do not effect the way other services are enjoyed, then  $C(y)$  just equals 1.

Consumer's problem is:

$$\begin{aligned} \max_{x, y_1, y_2, \dots} \quad & \alpha \log x + (1 - \alpha) \log [C(Y) \cdot (y_1^{\alpha_1^y} * \dots * y_N^{\alpha_N^y})] \\ \text{s.t.} \quad & x + p_1 * y_1 + p_2 * y_2 + \dots \leq I \end{aligned}$$

Assuming symmetry within sector  $Y$ , i.e.

$y_1 = y_2 = \dots = y$  and  $\alpha_1^y = \alpha_2^y = \dots = \frac{1}{N}$  and the price of combined service is  $P$ .

$$\begin{aligned} \max_{x, y} \quad & \alpha \log x + (1 - \alpha) \log [C(y) \cdot y] \\ \text{s.t.} \quad & x + P \cdot y \leq I \end{aligned}$$

FOC imply that consumer will choose an allocation such that:

$$\frac{(1-\alpha)}{C(y) \cdot y} * [C(y) + C'(y) \cdot y] = \frac{P \cdot \alpha}{x} \Rightarrow x^* = \frac{\alpha}{(1-\alpha)[1 + \frac{C'(y) \cdot y}{C(y)}]} * I$$

Therefore income share spent on sector  $x$ ,  $\alpha_x$  : 
$$\begin{cases} < \alpha; & \text{if } C'(y) > 0 \\ = \alpha; & \text{if } C'(y) = 0 \\ > \alpha; & \text{if } C'(y) < 0 \end{cases}$$

Because of above, service sector's income share increases if a service is introduced that is enhancing to existing services in the sector.  $C'(y)$  being positive means that the more services consumer consumes, more utility he gets from the services. Apart from service sector, another such example could be high tech electronics. If consumer consumes only digital camera, he gets limited utility. But if he consumes digital camera and computer and internet, his utility he gets from the same digital camera increases.

## 5.2 Actual Price of a Unit Util

A major technological discovery followed by its adoption in consumable services leads to major change in environmental variables in people's preferences. The previous allocation becomes non-optimal. Hence the consumer readjusts to the new optimal level.

At optimal allocation consumer equates  $\frac{MU_i}{p_i}$  for each sector. If one of the sector(service) innovates in such a way that its utility shifts up due to some externality or new innovation (Enhancing Complements), then at previous allocations  $\frac{MU_{service}}{p_{service}} > \frac{MU_{food}}{p_{food}}$ . Hence consumption of services increases. Income share spent on the service sector increases. Effectively the actual price of unit util goes down due to innovation.

If the product innovates in opposite way, that is it requires a lot of other services (Impeding Complements) to get any utility out of it, then income share drops. The logic is again similar. At optimal the MU from each sector should be equal for each dollar spent. To get any utility from the sector, consumer has to buy all of the products. If a new compliment is introduced, utility function shifts down OR effective price moves up.

## 5.3 Complementarity Parameter

We could think of  $C(y)$  as a function denoting the technological dependencies of various services in the sector. A simple functional form could be  $C(y) = y^{N_Y - 1}$ , where  $y$  is the index for services consumed and  $N_Y$  is the complementarity parameter for the sector. Notice that  $N_Y$  has to be between  $(0, \frac{1}{\alpha_Y})$  for utility function to follow usual properties.

$$\begin{cases} \text{if } \frac{1}{\alpha_Y} > N_Y > 1, & \text{then Enhancing Complements} \\ \text{if } 1 > N_Y > 0, & \text{then Impeding Complements} \end{cases}$$

1. For enhancing complements ( $N_Y > 1$ ) the utility function exhibits **Increasing return to scale** in Y sector.
2. For impeding complements ( $N_Y < 1$ ) the utility function exhibits **Decreasing return to scale** in Y sector.

In industry with low employment (like agriculture), there are not many Enhancing Complements. Hence share of the income spent does not grow. In new industries it is often the opposite.

## 6 Empirical Testing

The results suggested in this paper can be applied to various datasets and tested empirically.

Using datasets on features of service-offering for a particular industry, presence of time-constraint can be determine by testing whether:

1. More features are provided in single service.
2. Features being included in the service are complements rather than substitutes.
3. Transaction costs (time or effort it takes to obtain a service)are going down.

Similarly, using consumer expenditure data

1. If the service sector offers more complementary services rather than substitutes.
2. If the new innovations/ products in the sector are "enhancing" or "impeding" to existing products.
3. Sectors with different types (enhancing or impeding) innovation should see their income share change accordingly.